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Effective Density of Crumpled Paper Balls

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Abstract—*The physics of the crumpling of paper balls is quite complex and is not yet fully understood. Researchers have studied the 3D structure of crumpled paper balls using advanced experimental techniques like X-ray micro-tomography and discovered that crumpling is similar to other classic non-equilibrium problems like turbulence and very difficult to model mathematically. We have investigated how the diameter of paper balls obtained by crumpling square sheets of paper of different areas by hand, depends on the side-length of the paper. Our first model which assumed the density of the paper ball to be equal to the density of the paper used failed by predicting the wrong thickness of paper used. However, our second model which only equated the mass of the paper used to the mass of the paper ball obtained, and allowed for the effective density of the crumpled paper ball to be different from the density of paper used was successful in predicting the correct thickness of the paper and hence we concluded that the paper balls made by crushing by hand have a lower (around 3 times) constant effective density than the density of paper. If useful applications could be found for crushed paper balls which have many interesting properties like low density with high compressive strength and energy-absorbing properties, they could provide useful solutions for paper- waste management.*

Keywords*: paper ball, crumpling, effective density.*

1. INTRODUCTION

Academic institutions and corporate offices do a lot of paper work and generate a lot of paper waste. Recycling the paper by turning it back to pulp is one way of managing this waste. However, crumpled paper balls could also provide some alternative solutions for waste management since they have many interesting properties like high strength to weight characteristics, high compressive strength and high energy-absorbing properties [1] opening the possibilities for many innovative applications.

Earlier studies on crumpled paper balls have tried to model the 3D structure of crumpling using very advanced experimental techniques like X-ray micro-tomography [2] and laser-aided topography [3] or studied the shape, response and stability of conical dislocations, the simplest type of topological crumpling deformation [4]. Another study has established power-law dependence between the compaction force required to crumple a sheet into a ball and the diameter of the crumpled ball. In this study we investigate how the diameter of crumpled paper balls depends on the side-length of square sheets of paper used, by keeping the compaction force constant and equal to the maximum force that can be exerted by hand while crumpling the sheets of paper into a ball.

2. HYPOTHESIS

In the first hypothesis, we equated the volume of the sheet of paper to the volume of the paper ball by assuming the density of the crumpled paper balls to be equal to the density of the paper used.

$$l^2 t = \frac{4}{3}\pi\frac{d^3}{8} \qquad (1)$$

where l = length of side of square sheet of paper, t = thickness of sheet of paper, d = diameter of crumpled paper ball.

From equation (1), we obtain

$$d^3 = \left(\frac{6t}{\pi}\right) l^2 \qquad (2)$$

Thus, on plotting d^3 vs l^2, we should obtain a linear graph passing through the origin, and we should be able to calculate the thickness of the sheet of paper used from the slope of the graph. If it agrees with the actual thickness of the sheet of paper measured directly (0.00992 ± 0.00002 cm), then we can conclude that the hypothesis is correct.

In the second hypothesis, we equate only the mass of the sheet of paper to the mass of the paper ball.

$$\rho_p l^2 t = \rho_b \frac{4}{3}\pi\frac{d^3}{8} \qquad (3)$$

where ρ_p = measured density of paper sheet = 0.78 ± 0.02 $g.cm^{-3}$, ρ_b = effective density of crumpled paper ball.

From equation (3), we obtain

$$d^3 = \left(\frac{\rho_p}{\rho_b}\right)\left(\frac{6t}{\pi}\right) l^2 \qquad (4)$$

The second hypothesis predicts a smaller value for the slope of the linear fit if the effective density of the paper balls is lesser than the actual density of the sheets of paper used to make the crumpled paper balls. The thickness of the sheets of paper calculated using this value of slope should give a different value and thus can be used to identify the correct hypothesis by comparing it with the actual measured value of thickness of paper.

3. METHODS

Identical A3 sheets of paper were taken and cut into 40 square pieces (5 sets of eight different side-lengths). The square pieces of paper were crumpled into spherical paper balls by applying the maximum force possible by hand. The compacting force applied to make the crumpled paper balls was made constant in this way. The diameter of each paper ball was then measured using vernier caliper or screw gauge.

Figure 1. Measuring the diameter of crumpled paper balls

The density ρ_p of the sheets of paper used to make the crumpled paper balls was calculated to be 0.78 ± 0.02 gcm^{-3} by first measuring the mass of the biggest square piece of paper and dividing the mass by its volume. The thickness of a single sheet of paper, 't' was calculated to be 0.00992 ± 0.00002 cm by measuring the thickness of a pile of 48 sheets of paper using a screw gauge. The effective density of the crumpled paper ball for each diameter of crumpled paper ball was calculated using the mass per unit area and area of the sheet of paper used to make the paper ball, and the mean diameter of the crumpled paper ball.

4. RESULTS AND DISCUSSION

Figure 2 shows that the effective density of the crumpled paper balls does not depend on their diameter. The data point for the smallest paper ball is an outlier and the error could be due to the fact that its diameter was measured using a screw gauge whereas the diameters of all the other paper balls were measured using vernier calliper. The paper ball could have been crushed more during the process of measuring the mean diameter using screw gauge, causing the error.

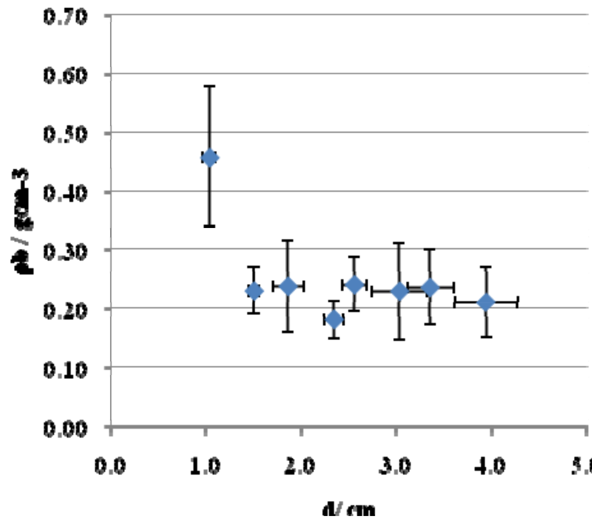

Figure 2 Effective density of crumpled paper ball vs mean diameter of paper ball

The mean effective density of the crumpled paper balls ρ_b was found to be 0.25 ± 0.06 gcm^{-3} which is much lesser than the density ρ_p of the sheets of paper used to make the crumpled paper balls.

Thus, hypothesis 1 has been proven to be invalid, since it assumed the density of the crumpled paper balls to be equal to the density of paper.

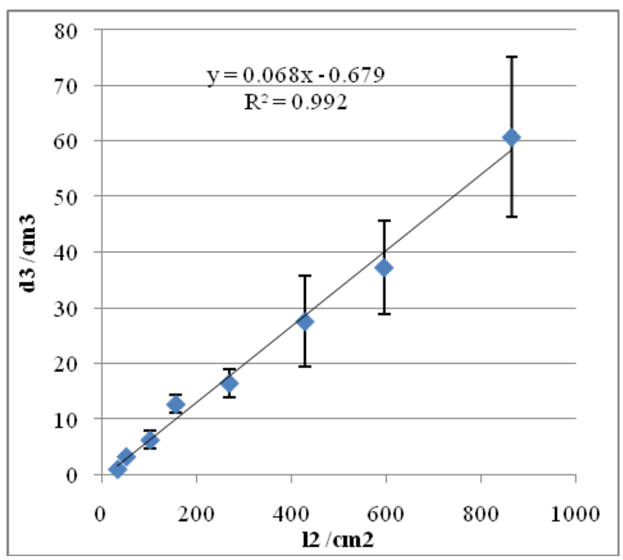

Figure 3 (diameter of crumpled paper ball)^3 vs (side-length of square sheet of paper)^2

Figure 3 shows that the linear best-fit straight line passes through the origin as predicted by hypothesis 2. Furthermore, the thickness of the sheet of paper calculated from the slope of the linear best-fit straight line by using the correcting factor $\frac{\rho_p}{\rho_b} = 3.09$ which takes into account the reduced effective density of the crumpled paper balls was found to be 0.012 ± 0.006 cm which is in perfect agreement with 0.00992 ± 0.00002 cm (the measured thickness of 1 sheet of paper). The uncertainty in the diameter of the crumpled paper balls was found to increase with the size of the paper balls because bigger paper balls would not acquire a perfectly spherical shape, and would compress more during the measurement process causing variation in the measured values for each trial.

For further research, a similar study could be conducted for balls made of strings or ribbons of different materials like wool, cotton, silk, etc by first soaking them in a liquid adhesive and then removing the excess adhesive by allowing it to drip away during compression. Their material properties like effective density, compressive strength and energy-absorbing properties could be studied.

5. CONCLUSION

The effective density of crumpled paper balls (0.25 ± 0.06 gcm^{-3}) was found to be around 3 times lesser than the density of paper (0.78 ± 0.02 gcm^{-3}) used to make them by compacting at maximum pressure that could be exerted by hand, for square sheets of paper of side-lengths ranging from 6 cm to 30 cm. The effective density of crumpled paper balls does not depend on the area of paper used or the diameter of the crumpled paper balls formed. It depends only on the maximum compacting force applied to crumple and make the paper balls, which was a constant in this investigation and equal to the maximum force that could be exerted by hand.

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